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TRACOR Document No.

SD-67-0009-U

Contract (N123(953)54996A).

TRACOR Project No. 002-009-23

(12) 13p.

(14) TRACOR-SD-67-\$\$\phi\$\$\phi\$99-U-ADD

TECHNICAL NOTE,

**ADDENDUM** 

DETERMINATION OF SHIP COURSE SPEED

FROM TRANSMITTED SIGNAL FREQUENCY.

Submitted To

Conformal/Planar Array
Project Office
Code 2110 - USNEL

11 24 Apr 67

ACCESSION for

HTTE Notice Selection Selection

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#### 1. INTRODUCTION

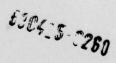
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This report supplements TRACOR Report No. SD-67-0007-C, "Determination of Ship Course and Speed from Transmitted Signal Frequency", dated 22 March 1967. The work of that report is extended here to cover the following two additional cases:

- 1. Surface vessel transmitting with full-doppler nullification but with broad beams instead of narrow;
- 2. Surface vessel transmitting with half-doppler nullification and submarine utilizing side-lobe frequency information.

It is shown in this report that the submarine will be able to calculate surface vessel course and speed when the surface vessel is transmitting broad-beam full-doppler nullified signals. An additional observation of frequency and bearing is required of the submarine over the narrow-beam case.

The utilization of side-lobe frequency information has been found to make it possible for the submarine to calculate surface vessel course and speed in the half-doppler nullification case. Equations have been derived for these course and speed values for the case of narrow-beam transmissions. It is shown that solution is theoretically possible even in the broad-beam case.



### 2. BROAD BEAM SIGNALS WITH FULL-DOPPLER NULLIFICATION

An analysis, given in the original report, shows that when a surface vessel is known to be pinging with full-doppler nullification, a submarine can calculate the surface vessel's course and speed from two observations of the bearing and frequency of the incoming signal. This analysis, however, was based on the assumption that the surface vessel was transmitting narrow beam pulses and that all observations of signal frequency were made when the narrow beam was aimed at the submarine.

The use of broad transmit beams by the surface vessel will require modification of this analysis. The frequency at which the surface vessel drives the array will remain

$$f_D = \frac{f_c}{1 + \frac{2v}{c} \cos \theta_o}$$

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where  $\theta_0$  is the centerbeam direction. The frequency into the water, however, will become

$$f_{w} = \frac{f_{c}}{1 + \frac{2v}{c} \cos \theta_{o}} \left[ 1 + \frac{v}{c} \cos \theta \right]$$

or 
$$f_w \sim f_c \left[ 1 + \frac{v}{c} \cos \theta - \frac{2v}{c} \cos \theta_o \right]$$

where  $\theta$  is any direction of radiation under consideration and may be the same as or different from  $\theta_0$ , the centerbeam direction. With this signal in the water, the submarine will now hear

$$f_{RS} \sim f_c \left[1 + \frac{v}{c} \cos \theta - \frac{2v}{c} \cos \theta_o\right] \left[1 + \frac{v_s}{c} \cos \phi\right]$$

or 
$$f_{RS} \approx f_c \left[ 1 + \frac{v}{c} \cos \theta - \frac{2v}{c} \cos \theta_0 + \frac{v}{c} \cos \phi \right]$$

This equation is similar to that found for the narrow beam case, but does involve one additional unknown,  $\theta_0$ , the centerbeam direction. If observations of signal frequency and bearing to the surface ship are made by the submarine three times now, instead of two, five equations with five unknowns  $(\theta_1, \theta_2, \theta_3, \theta_0, v)$  will result instead of the previous three equations and three unknowns  $(\theta_1, \theta_2, v)$ . This is on the assumption that the submarine knows that the centerbeam direction,  $\theta_0$ , is the same for all three observations. If the surface vessel has a regular azimuthal sweep routine and the submarine makes its observations on successive sweeps, this seems to be a realistic assumption.

It appears, then, that it will be possible for the submarine to determine the surface vessel's course and speed even when the surface vessel is transmitting broad beams. However, no attempt has been made here to solve the five equations and five unknowns in order to arrive at the equations for course and speed in terms of the frequency and bearing measurements which could be made by the submarine. It has been considered sufficient to demonstrate that solution is still theoretically possible.

3. SPEED AND COURSE DETERMINATION WITH HALF-DOPPLER NULLIFICATION USING SIDELOBE FREQUENCY INFORMATION

For half-doppler nullification, the frequency at which the surface vessel drives the transmitting array is given by

$$f_{D} = \frac{f_{C}}{1 + \frac{v}{c} \cos \theta_{O}} \tag{1}$$

where f<sub>c</sub> = the frequency to which all centerbeam transmissions into the water are held

v = own-ship's speed

c = speed of sound in water (4900 ft/sec)

 $\theta_0$  = centerbeam direction

The frequency transmitted into the water in any direction  $\theta$ , which may be on or off the centerbeam direction, is given by

$$f_{w} = \frac{f_{c}}{1 + \frac{v}{c} \cos \theta_{c}} \left[ 1 + \frac{v}{c} \cos \theta \right]$$
 (2)

or

$$f_w \approx f_c \left[ 1 + \frac{v}{c} \left( \cos \theta - \cos \theta_0 \right) \right]$$
 (3)

For the centerbeam direction,  $\theta$  is equal to  $\theta_{\mbox{\scriptsize 0}}$  and the equation reduces to

$$f_w = f_c \tag{4}$$

The frequency of the signal received by the submarine which lies in the direction  $\boldsymbol{\theta}_{\mathbf{S}}$  is given by

$$f_{RS} \approx f_c \left[1 + \frac{v}{c} \left(\cos \theta_s - \cos \theta_o\right)\right] \left[1 + \frac{v_s}{c} \cos \phi\right]$$
 (5)

or 
$$f_{RS} \approx f_c \left[ 1 + \frac{v}{c} \left( \cos \theta_s - \cos \theta_o \right) + \frac{v_s}{c} \cos \phi \right]$$
 (6)

Solving for f

$$f_{c} = \frac{f_{RS}}{1 + \frac{v}{c} (\cos \theta_{s} - \cos \theta_{o}) + \frac{v_{s}}{c} \cos \phi}$$
 (7)

If the submarine lies on the centerbeam,  $\theta_s$  equals  $\theta_0$  and the equation reduces to

$$f_c = \frac{f_{RS}}{1 + \frac{v_s}{s} \cos \varphi} \tag{8}$$

The submarine will always know from the relative intensity of the received signal when it lies within the transmitted beam. If the surface vessel is transmitting narrow beams,  $\theta_s$  and  $\theta_o$  will always be nearly equal when the beam is aimed toward the submarine. Thus, under these circumstances, the submarine will be able to calculate  $f_c$  from the frequency,  $f_{RS}$ , observed when the beam is aimed at the submarine, together with readings of its own speed,  $v_s$ , and the relative bearing to the surface vessel,  $\varphi$ . Solving for  $f_c$  in this manner reduces the number of unknowns in Eq. (6) from four to three  $(v, \theta_s, and \theta_o)$ .

By making observations at three separate times,  $t_1$ ,  $t_2$ , and  $t_3$ , three separate equations can be written as follows

$$f_{RS1} = f_c \left[ 1 + \frac{v}{c} (\cos \theta_1 - \cos \theta_0) + \frac{v}{c} \cos \phi_1 \right]$$
 (9)

$$f_{RS2} = f_c \left[ 1 + \frac{v}{c} (\cos \theta_2 - \cos \theta_0) + \frac{v_s}{c} \cos \phi_2 \right]$$
 (10)

$$f_{RS3} = f_c \left[ 1 + \frac{v}{c} (\cos \theta_3 - \cos \theta_0) + \frac{v_s}{c} \cos \phi_3 \right]$$
 (11)

where  $f_{RS1}$ ,  $f_{RS2}$ , and  $f_{RS3}$  are the frequencies, and  $\phi_1$ ,  $\phi_2$ , and  $\phi_3$  are the bearings observed by the submarine at times  $t_1$ ,  $t_2$ , and  $t_3$ , and  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  are the bearings from the surface vessel to the submarine which exist at those times. The centerbeam direction,  $\theta_0$ , and the speed of the surface vessel are assumed to be of the same unknown value in all three equations. Thus, the submarine now has three equations and five unknowns,  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$  as well as  $\theta_0$  and v.

The change in bearing from the submarine to the surface vessel must always be equal to the change in bearing from the surface vessel to the submarine. Therefore,

$$\theta_2 - \theta_1 = \varphi_2 - \varphi_1 \tag{12}$$

$$\theta_3 - \theta_2 = \varphi_3 - \varphi_2 \tag{13}$$

or 
$$\theta_2 = \theta_1 - \Delta \phi_{21}$$
 where  $\Delta \phi_{21} = \phi_2 - \phi_1$  (14)

and 
$$\theta_3 = \theta_2 - \Delta \phi_{32}$$
 where  $\Delta \phi_{32} = \phi_3 - \phi_2$  (15)

This gives two more equations but no more unknowns. Thus, the submarine has five equations and five unknowns.

Solution of these equations shows that

$$\alpha = \beta + \phi_1 - \pi - \tan \theta_1 \tag{16}$$

$$v = \frac{K_1 - K_2}{\cos \theta_1 - \cos(\theta_1 + \Delta \phi_{21})}$$
 (17)

where 
$$\theta_1 = \tan^{-1} \frac{K_3 - K_2 + (K_1 - K_3) \cos \Delta \phi_{21} - (K_1 - K_2) \cos \Delta \phi_{31}}{(K_1 - K_3) \sin \Delta \phi_{21} - (K_1 - K_2) \sin \Delta \phi_{31}}$$
 (18)

$$K_1 = \left(\frac{f_{RS1}}{f_c} - 1\right) c - v_s \cos \varphi_1 \tag{19}$$

$$K_2 = \left(\frac{f_{RS2}}{f_c} - 1\right) c - v_s \cos \varphi_2 \tag{20}$$

$$K_3 = \left(\frac{f_{RS3}}{f_c} - 1\right) c - v_s \cos \varphi_3 \tag{21}$$

with  $\alpha$  = surface ship's heading

v = surface ship's velocity

s = submarine's heading

v<sub>s</sub> = submarine's velocity

 $φ_1$  = submarine to ship bearing at time  $t_1$   $φ_2$  = submarine to ship bearing at time  $t_2$   $φ_3$  = submarine to ship bearing at time  $t_3$   $f_{RS1}$  = frequency measured by submarine at time  $t_1$   $f_{RS2}$  = frequency measured by submarine at time  $t_2$   $f_{RS3}$  = frequency measured by submarine at time  $t_3$   $θ_1$  = ship to submarine bearing at time  $t_1$   $θ_2$  = ship to submarine bearing at time  $t_2$   $θ_3$  = ship to submarine bearing at time  $t_3$   $Δφ_{21}$  =  $φ_2$  -  $φ_1$   $Δφ_{31}$  =  $φ_3$  -  $φ_1$  π =  $180^{\circ}$ 

In this solution, it is assumed that the relative direction of training of the transmit beam,  $\theta_0$ , is the same for all three observation times, t1, t2, and t3. The submarine has no direct way of knowing in what direction the surface vessel is pinging at any time except when the beam is aimed at the submarine. Even then it is only known to within the beamwidth. But observations made when the beam is aimed at the submarine are useful only in determining  $f_c$ . Solutions for v, and  $\alpha$  require readings of  $\boldsymbol{f}_{RS}$  and  $\boldsymbol{\phi}$  when the beam is not aimed at the submarine. If the surface vessel is sweeping in a regular manner, the frequency of the sound the submarine hears will shift in a regular manner from pulse to pulse. Thus, the maximum and minimum frequencies would correspond to the azimuthal extremes in direction. surface vessel were sweeping consistently between the same extremes, the submarine would know when the beam was aimed in those directions. By counting pulses from those extremes any other direction could serve as the reference. The solution does

not require knowing what the direction is, but merely that all observations be made when the beam is aimed in the same direction.

If the surface vessel were transmitting broad beams,  $\theta_s$  and  $\theta_o$  would not necessarily be equal in Eq. (7) and it would not be possible for the submarine to solve accurately for  $f_c$ . In that case, a fourth observation could be made which would add two more equations to the five in (9) through (13)

$$f_{RS4} = f_c \left[ 1 + \frac{v}{c} (\cos \theta_4 - \cos \theta_0) + \frac{v_s}{c} \cos \phi_4 \right]$$

$$\theta_4 = \theta_1 + \Delta \phi_{41}$$

This would add  $\theta_4$  as an unknown, and if  $f_c$  were now considered to be unknown, the submarine would have seven equations and seven unknowns. This should make it possible for the submarine to solve for surface vessel course and speed in the broad transmitbeam case. No attempt has been made here to evolve this solution.

#### 4. CONCLUSIONS

- 1. It will be possible for a submarine to determine the course and speed of a surface vessel which is transmitting broad-beam full-doppler nullified signals. An additional observation of signal frequency and bearing to the surface vessel will be required over the narrow-beam case.
- 2. It will be possible for a submarine to determine the course and speed of a surface vessel which is transmitting half-doppler nullified signals. In the narrow-beam case, one centerbeam and three side-lobe observations of frequency and bearing are required. For broad-beam transmissions a fourth side-lobe reading of frequency and bearing is required in place of the centerbeam readings.
- 3. Neither full-doppler nor half-doppler nullification reveal any more about the surface vessel's course and speed than does ordinary no-doppler nullification.